Journal of Global Optimization **12:** 197–214, 1998.

A Multiple Criteria Analysis Model For Real Estate Evaluation

OSSAMA KETTANI[1], MUHITTIN ORAL[2] and YANNIS SISKOS[3]
[1]*Faculté des Sciences de l'administration, Université Laval, Ste-Foy (Québec), G1K 7P4, Canada. (Email: Ossama.Kettani@osd.ulaval.ca) Tel.: (418) 656-3148; Fax: (418) 656-7722.*
[2]*Graduate School of Future Management, Sabanci University, Istanbul, Turkey*
[3]*Technical University of Crete, Decision Support Systems Laboratory University Campus, 7310 Chania, Greece.*

(Received and accepted: 2 May 1997)

Abstract. Real estate evaluation is of great importance and interest to many socio-economic agents, especially to property buyers and sellers for personal benefits, municipalities for tax purposes, financial institutions for loan policies, and to real estate brokerage firms for marketing activities. Although these agents are motivated in their actions by different objectives, even conflicting at times, they all desire to have a realistic description of the real estate market behavior in order to make right and timely decisions. This article presents an estimation model to describe the behavior of real estate markets. The model is based on certain observable real estate market data as well as on the perceptions of real estate agents who are active in the market. The parameters that describe the behavior of the real estate market are estimated, through the estimation model, using mathematical programming tools within a multiple criteria analysis context. The usefulness and applicability of the approach is empirically shown through an implementation using the data of the City of Edmonton, Alberta, Canada.

Key words: Real estate appraisal, estimation methods, ordinal regression, mathematical programming, multiple criteria approach.

1. Introduction

Real estate evaluation is an activity by which the value of real estate is estimated in monetary terms to support a large variety of decisions and policies such as buying, selling, mortgaging, and determining annual property taxes. In fact, real estate evaluation is an important area of interest to many socio-economic agents particularly to municipalities for taxing purposes, financial institutions for granting loans, real estate brokerage firms for marketing activities, and of course the individuals and the companies involved as buyers or sellers in real estate transactions. In this paper, we have only considered the market of *residential properties*. Below is a short view of the nature of the implication of each agent related to this market, regarding real estate evaluation activity.

i. Municipalities, which revenues depend largely on residential real estate taxes, establish their own real estate evaluation, generally with the intermediate of a local service aimed towards that objective. On one hand, they are concerned to preserve and increase that source of revenue in a reasonable and justifiable way; and on the other hand, to assure a certain level of fairness and equity in their evaluations in the hope of reducing or minimizing the number of complaints and dissatisfactions against the municipality taxes charged.

ii. Financial institutions, where a large portion of financial activities is related to housing credits, need to have the values of real estate properties in order to grant mortgage loans to their clients without running into high risks. Financial institutions usually obtain such real estates values from private evaluators or real estate brokerage firms.

iii. Real estate brokerage firms and private evaluators, who are usually in the center of most property transactions, need to evaluate real estate properties for their clients to help them take judicious decisions.

iv. Residential property buyers and sellers are engaged, in a more or less formal way either directly or with the intermediate of a third party (private evaluators, real estate brokerage firms, etc.), in real estate evaluation in order to make most rational decisions and hope to gain most from such transactions.

Although these agents, who are involved in real estate evaluation in one way or another, are motivated in their actions to pursue different objectives, they all desire to make or obtain a reliable and realistic evaluation of real estate market in order to make right and timely decisions. The quality of such evaluations however depends mostly on the perception of these agents regarding the market behavior of residential properties in particular and the socio-economic environment in general. The first objective of this paper is precisely to construct a descriptive model, as realistic as possible, of this market behavior that permits to constitute a basis for real estate evaluation.

In this paper, it is assumed that the preferences in residential property market are ultimately surrogated in property selling prices. Although selling price is taken as the surrogate for market preferences there is a need for a better understanding how a selling price of a property emerges. There are several factors at work in property markets. Number of rooms, garages, fireplaces, size of residence are but few examples. Therefore the selling price of a property can be considered as a function of its characteristics or attributes. In this perspective, the market behavior can be modeled as a multicriteria choice process in which residential property market expresses its preferences by assigning to each property a selling price, which is itself a function of the attributes of the property in question.

The real estate evaluation model suggested in this paper employs multiple criteria analysis approach and uses selling prices and observable residential property attributes as the data base. Also imbedded in the construction of the model is the flexibility of integrating the expertise and experience of real estate agents to make evaluation more realistic and interactive. This feature of the model offers an avenue of flexible applications to meet a variety of needs in the implementation process.

The estimation model suggested in this paper is in essence a descriptive multiple criteria analysis model. Descriptive multiple criteria approaches can be classified into two groups: those based on statistical techniques and those based on mathematical programming. Statistical methods include, particularly, regression analysis [1], monotonous regression [3] and multidimensional scale analysis [2] whereas mathematical programming methods are represented particularly by the works of Srinivasan and Shocker [18, 19], Pekelman and Sen [13], Jacquet-Lagrèze and Siskos [5], Siskos and Zopounidis [15] and Stewart [17]. Many studies converge to establish the superiority of approaches based on mathematical programming, notably linear programming, in terms of their predicted power [4], Shocker and Srinivasan [16], Jain et al. [6] and Siskos [15]. The estimation method proposed in this paper belongs to the second group and constitutes an extension of the multi-criteria analysis methods of Kettani [8], Oral and Kettani [10, 11, 12] and Oral et al. [12].

The organization of this paper is as follows: Section 2 presents the context in which the empirical real estimation problem is defined and introduces the notation needed. Section 3 includes the details of constructing the estimation model. Two variants of the estimation model are given in Section 4. Section 5 is devoted to the empirical study done using the data of the City of Edmonton, Alberta, Canada. Section 6 concludes the paper with some remarks.

2. Context of the Empirical Study

Let $A = \{i|1 \leq i \leq m\}$ be a representative sample of residential properties taken from a population of real estates and includes m properties. Also let $C = \{j|1 \leq j \leq n\}$ be a set of n criteria considered to be the most pertinent in the process through which the real estate market evaluates the properties and assign a selling price to each one of them. Moreover, let V_{ij} be the value or score of property i with respect to criterion j, and p_i be the observed selling price for this property. Furthermore, we assume that $V_{ij} \geq 0$.

The empirical study is based on the data obtained from the City of Edmonton, Alberta, Canada. The estimation model is used to describe the market behavior of bungalow sales in Edmonton. For this purpose, all 133 properties of this category sold during the year of this study are identified. However, 25 of these properties are dismissed because they display extreme value cases or outliers with respect to one criterion or another. Thus, the set **A** has 108 observations. For this set, sale price varies between \$79,100 and \$144,500 and the average sale price is \$107,171.

At the end of a consulting process with a group of real estate agents, it has been concluded that eleven criteria were the most pertinent ones governing the behavior of the bungalow market. Below are these criteria and their brief descriptions:

Criterion 1. *Age*: measured in years since the date of construction. The observed values (V_{i1}) of the bungalows in the sample with respect to this criterion vary between 14 and 28 years.

Criterion 2. *Size*: measured in square meters of floor area. The observed values (V_{i2}) in the sample with respect to this criterion vary between 90 and almost 150 square meters.

Criterion 3. *Number of garages*: The number of garages (V_{i3}) in the sample are 0, 1 or 2.

Criterion 4. *Easy access to the garage*: It is considered that there is an easy access if the garage is attached to the bungalow, otherwise it is considered not an easy access. Therefore, V_{i4} will take on value 1 if the garage is attached to the bungalow, 0 otherwise.

Criterion 5. *Basement*: This criterion is used to indicate to which extent the basement of a bungalow is completed. Given the fact that all bungalows in the sample possess a basement, the value of 1 implies that the basement is not finished, 2 means the basement is half-finished, and 3 means that the basement is entirely finished.

Criterion 6. *Fireplace*: This criterion is to indicate the number of fireplaces in a bungalow. It was observed that the number of fireplaces in a bungalow in the sample were 0, 1 or 2.

Criterion $6+k$. *Sector Identification*: The bungalows in the sample are located in five different sectors of the city. In order to identify the sector in which a bungalow is located, the sector identification criterion is introduced. If a bungalow is located in Sector k then a value of 1 is assigned, 0 otherwise. The sector indicator k takes on values 1, 2, 3, 4, and 5 to represent five residential sectors in Edmonton, namely: (1) Greenfields and Petrolia, (2) Royal Gardens, (3) Lendrum and Malmo, (4) Aspen, and (5) Rideau Park and Shaughnessy.

The data of this empirical study are given in Appendix 1 in terms of the values $V_{ij}, i \in \mathbf{A}$ and $j \in \mathbf{C}$. In order to reduce space, the values $V_{ij}, j = 7, \ldots, 11$, are presented in an implicit way in only one column. That way V_{ik} equals 1, if the indication $(j-6)$ appears in this column at line i, or 0 otherwise.

3. Modeling Market Behavior

In modeling the behavior of bungalow market it is assumed that multiple criteria choice process is the underlying process of market behavior and the sale price p_i is the surrogate for the market's preference regarding property i. This preference is assumed, in return, to be the result of the joint attraction of the property's criteria, presented by $V_{ij}, j \in C$.

The multiple criteria choice process that determines sale price, can be assumed to realize itself in two steps: a "local" evaluation step where the individual contribution of each criterion to the formation of selling price, and a "global" evaluation step where the individual contributions obtained in the first step are integrated into a whole in order to determine the selling price. These two steps are briefly described below.

3.1. "LOCAL" EVALUATION STEP

It is conceivable to assume that the scores obtained with respect to each criterion contribute, in monetary terms, to the sale price of a bungalow. The contribution of the criterion j to the selling price of the property i is called *marginal contribution* and denoted by U_{ij}. Such a contribution may be positive or negative one. For example, the criterion ≪floor area≫ provides a positive contribution whereas the criterion ≪age≫ has a negative impact on the selling price. We will suppose that U_{ij} is obtained as follows:

$$U_{ij} = f_j(V_{ij}). \tag{1}$$

where f_j is the *local evaluation function* of the criterion j. To illustrate, if the criterion j corresponds to the ≪floor area≫, and equals to 120 square meters, and if the marginal contribution is given by $f_j(V_{ij}) = 500V_{ij}$ then the marginal contribution U_{ij} is \$60,000. In the formula used, the rate of contribution is assumed to be constant and equal to \$500 per square meter. This implies that marginal contribution function is linear. This may not be the case all the time. Therefore one needs to consider functions other than linear. In fact, this is exactly what has been considered in the formulation of estimation model to reflect a variety of functional forms for marginal contribution.

In this paper, two functional forms are considered: (1) piecewise linear function, and (2) monotonous nonlinear function. These are explained below.

3.2. PIECEWISE LINEAR LOCAL EVALUATION FUNCTION f_j:

Subdividing the variation interval of possible non null values of the criterion j into T_j mutually exclusive and collectively exhaustive intervals (the index j in T_j will be left out when there is no risk of ambiguity) and noting that D^-_{tj} and D^+_{tj} the interval's lower and upper limits and t, $1 \le t \le T_j$, such that

$$D^+_{tj} \le D^-_{t+1}, \quad \text{for } 1 \le t \le T_j - 1.$$

Now, we can define f_j as follows:

$$f_j(V) = \begin{cases} 0, & \text{if } V = 0 \\ a_{tj} + VW_{tj}, & \text{if } V \neq 0 \text{ and } D^-_{tj} \le V \le D^+_{tj} \end{cases} \tag{2}$$

where a_{tj} and W_{tj} are parameters, from the piecewise linear, belonging to t-th interval of the criterion j and where V is any value included between D^-_{tj} and D^+_{tj}.

The decomposition into subintervals to have a piecewise linear representation of marginal contribution function depends not only on the nature of the criterion in

Table 1. Intervals of Piecewise Linearization for the Criteria Used.

	Number of intervals	Range of intervals
Criterion 1	5	[14, 16.8); [16.8, 19.6) [19.6, 22.4); [22.4, 25.2) [25.2, 28]
Criterion 2	5	[900, 1017); [1017, 1134) [1134, 1250); [1250, 1367) [1367, 1484]
Criterion 3	2	[1,1]; [2,2]
Criterion 4	1	[1,1]
Criterion 5	3	[1,1]; [2,2]; [3,3]
Criterion 6	3	[0,0]; [1,1]; [2,2]
Criterion 7–11	1	[0,1]

question but also on how sensitive it is perceived the selling price to the variations in the value obtained with respect to this criterion. It is important to determine subintervals in such a way that this kind of sensitivity is captured in piecewise linearization. The other point to be kept in mind is that there must be enough numbers of observations in each subinterval in order to have a reliable estimation. In case of small number of discrete values, on the other hand, one might consider each value as an interval by itself. In fact, this is the case for the criteria 3 to 11 in Table 1 and are treated as such in the empirical study. See Table 1 for the interval subdivisions for each criterion.

3.3. Monotonous Local Evaluation Function f_j

Assuming that marginal contribution is a monotonous function, be non-decreasing or non-increasing, is not restrictive in the sense that a change in the value of a bungalow with respect to certain criteria could have a negative impact and with respect to others a positive one. Let us decompose the criterion set C in two mutually exclusive and collectively exhaustive sets C^- and C^+, where C^- is the subset of those criteria which are associated with monotonous non-increasing functions, and C^+ is the subset of those criteria which are associated with monotonous non-decreasing functions. In mathematical notations, we have:

$$f(D_{tj}^+) \leq f(D_{t+1,j}^-), \quad \text{for } 1 \leq t \leq T_j - 1, j \in C^+ \tag{3}$$

$$W_{tj} \geq 0, \quad \text{for } 1 \leq t \leq T_j,\ j \in C^+ \tag{4}$$

$$f(D_{tj}^{+}) \geq f(D_{t+1,j}^{-}), \quad \text{for } 1 \leq t \leq T_j - 1,\ j \in C^{-} \tag{5}$$

$$W_{tj} \leq 0, \quad \text{for } 1 \leq t \leq T_j,\ j \in C^{-} \tag{6}$$

In this empirical study, it was concluded that the local evaluation function related to the criterion $\ll$age$\gg$ is monotonous non increasing, because, all other things being the same, a new bungalow is preferred to an old one. On the other hand, it was concluded that the local evaluation functions associated with the rest of the other criteria are monotonous non decreasing. Hence, $C^{-} = \{1\}$ and $C^{+} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

3.4. Global Evaluation Step

This step consists of integrating the marginal contributions obtained with respect to each criterion into a total or global value, which is an estimate for the selling price of a bungalow. It is assumed that the marginal contributions U_{ij} explain p_i, the property's selling price i through

$$p_i \cong h(U_{ij}, \ldots, U_{in}), \tag{7}$$

where h is the global evaluation function that allows to estimate the sale price. The analytical form of h is assumed to be

$$h(U_{i1}, \ldots, U_{in}) = U_0 + \sum_{j \in C} U_{ij} \tag{8}$$

where U_0 represents a constant value for a property when all the marginal contributions are nonexistent. U_0 will be henceforth called $\ll$basic contribution$\gg$. The additive form assumed for h implies that the criteria are *preferentially* independent from one another. For preferential independence in the context of utility theory and other underlying conditions, the reader is referred to Kenney and Raïffa [7]. If the preferential independence assumption does not hold, the global evaluation function h should be expressed under other alternative forms. Notice that testing this independence hypothesis is not a trivial task. However, by defining the criteria in an appropriate manner, one can often minimize the risk of dependence.

4. Estimation Model

The estimation model, which is a mathematical programming model, aims at describing the market behavior of bungalows through estimating the parameters of the global evaluation function h and the local evaluation functions f_j, namely U_0, and a_{tj} and $W_{tj}, j = 1, \ldots, T_j$. Below is the first variant of the estimation model, which will be called (M_A).

(M_A) :

$$\text{Minimize } D_A = \sum_{i \in A} 1/p_i(d_i^- + d_i^+) \tag{9}$$

subject to

$$U_0 + \sum_{j \in C} U_{ij} - d_i^+ + d_i^- = p_i, d_i^- \geq 0, d_i^+ \geq 0, \quad \text{for } i \in A \tag{10}$$

$(3), (4), (5)$ and (6)

where U_{ij} is as defined in (1), f_j as defined in (2), and d_i^- and d_i^+ represent, respectively, the negative and positive deviation between p_i, the observed selling price, and $U_0 + \sum_{j \in C} U_{ij}$, the estimated selling price for property i. Some remarks are in order for the variant (M_A):

(i) First, its use requires the construction of the sets A, C, C^- and C^+, to collect the values $V_{ij}, i \in A, j \in C$, and the sale prices $p_i, i \in A$, and to determine for each criterion j, $j \in C$, the number of intervals T_j and the lower and upper limits D_{tj}^- and D_{tj}^+ for each interval t, $t = 1, \ldots, T_j$.

(ii) Second, we note that the expression $1/p_i(d_i^- + d_i^+)$ in (9) represents the absolute deviation rate for the estimated selling price $(U_0 + \sum_{j \in C} U_{ij})$ of the property i compared to its observed selling price (p_i).

(iii) Finally, the average deviation rate

$$\mu_A = D_A/m \tag{11}$$

which is the consequence of "unfitness" of (M_A) to the market behavior data, is not entirely a perfect measure for the reliability of the model. In other words, a low level of μ_A does not necessarily guarantee that the individual variations will also be low. There is always a possibility that one might end up with high individual deviations for some bungalows even when the average deviation is low.

To overcome this difficulty, we can consider a second variant of the estimation model, called (M_B):

(M_B) :

$$\text{Minimize } D_B = \sum_{i \in C} 1/p_i(d_i^- + d_i^+) \tag{12}$$

subject to

$$1/p_i(d_i^- + d_i^+) \leq e, \quad \text{for } i \in A \tag{13}$$

$(10), (3), (4), (5)$ and (6)

This variant differs from (M_A) because of the additional constraints (13), which impose the maximum acceptable deviation rate (in absolute value) represented by e. Of course, these constraints might not be feasible if e is under a certain level, $\underline{e}$. In fact, (13) will be feasible if, and only if,

$$e \geq \underline{e} \tag{14}$$

$$\text{where } \underline{e} = \mathop{\text{Min}}_i(\text{Max}(1/p_i(d_i^- + d_i^+))) \quad \text{subject to (10), (3), (4), (5) and (6)} \tag{15}$$

As (15) indicates, this threshold value $\underline{e}$ is the lowest possible level of the absolute value of the maximum deviation rate. In an equivalent way to (15), this threshold can be obtained from the following linear program (P):

$$
\begin{aligned}
&(P):\\
&\qquad \underline{e} = \text{Min } z\\
&\qquad \text{subject to}\\
&\qquad 1/p_i(d_i^- + d_i^+) \leq z, \quad \text{for } i \in A\\
&\qquad (10), (3), (4), (5) \text{ and } (6)
\end{aligned}
$$

Let

$$\mu_B = D_B/m \tag{16}$$

be the average deviation rate resulting from (M_B). We immediately note that μ_B cannot be equal or greater than μ_A. In fact, opting for variant (M_A) or for variant (M_B), the estimation method becomes a choice between a minimal value of the average deviation rate or a minimal value of the maximum deviation rates.

Regardless of the choice, be the variant (M_A) or the variant (M_B), one can formulate constraints to reflect the information *a priori* available on the bungalow market behavior. For example, one can impose a lower limit U_j^- and an upper limit U_j^+ on the marginal contribution with respect to a criterion, say j. In mathematical notations,

$$f(D_1^-) \geq U_j^-, f(D_T^+) \leq U_j^+ \text{ if } j \in C^+ \tag{17}$$

$$f(D_T^+) \geq U_j^-, f(D_1^-) \leq U_j^+ \text{ if } j \in C^- \tag{18}$$

We can also consider other constraints on the parameters U_o, W_{ij} and a_{tj} in order to reflect particular requirements for the global and local evaluation functions.

5. The Results of the Empirical Study

In this section we shall present and discuss the results obtained by using successively the variants (M_A) and (M_B) of the estimation method. It is important to mention here that the objective of this section is only to illustrate numerically the estimation method. In other words, the interest and the interpretation of the results in the particular field of real estate evaluation is not the main goal. Also, notice that most of the mathematical programming software can be used to solve the linear programs (M_A) and (M_B). In this empirical study, we have used LINDO.

As can be observed in Appendix 2, the optimal solution obtained from the variant (M_A) gives a basic contribution of \$70,365. To this basic value one needs to add individual marginal contributions with respect to every single criterion to obtain the selling price of the bungalow. Also shown in Appendix 2 are the

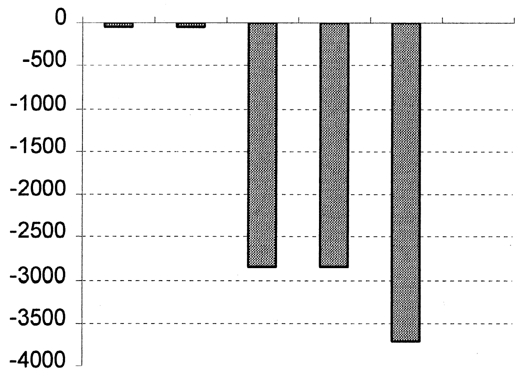

Fig. 1. Marginal Contribution of Age

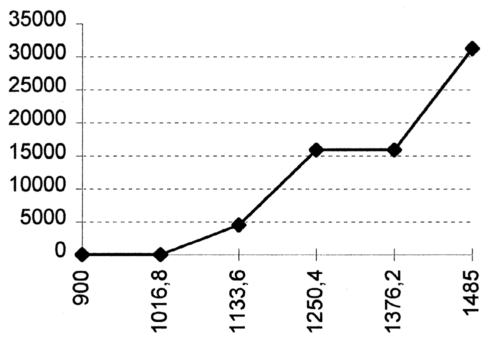

Fig. 2. Marginal Contribution of Size

marginal contribution rates with respect to each criterion and the intervals for which these rates are valid. For instance, take the first criterion, which is the “age” of bungalows (see Figure 1). One can immediately observe that there are three groups of intervals. In the first group (interval 1 and 2), the marginal contributions are zero. In the second group (interval 3 and 4), the marginal contributions are negative and the same ($2,857.57), and in the third group (interval 5), it is again negative but this time $3,724.82. The implication of this observation is that one in fact needs three intervals, not five as initially determined prior to the estimation process. These three intervals are [14, 19.6), [19.6, 25.2) and [25.2, 28].

Consider now the second criterion: floor area. Initially there were five intervals and the model also suggests and uses the same five intervals, for they all have different marginal contributions in the said intervals. Also observe that marginal contribution function is a monotonous non-decreasing one, for the second criterion is included in C^+. Finally, notice that the marginal contributions in the first and fourth interval are constant whereas they are proportional in the other intervals. Figure 2 shows the behavior of local evaluation function of floor area per interval.

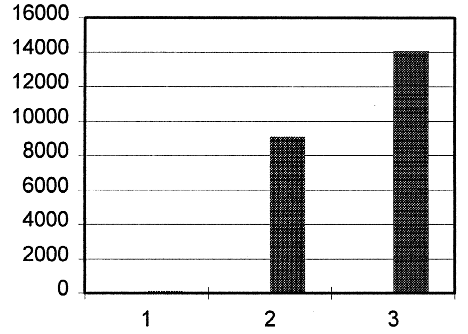

Fig. 3. Marginal Contribution of Basement

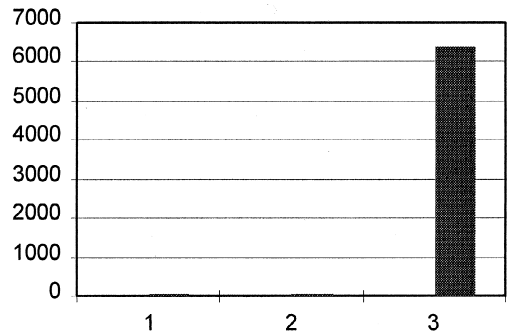

Fig. 4. Marginal Contribution of Fireplace

Contrary to the two previous local evaluation functions, which were defined in continuous intervals, the rest of local evaluation functions were defined on discrete scales (refer to Table 1). Take, for instance, the third criterion: the number of garages. From Appendix 2, we conclude that the marginal contribution of one garage is \$8,390.42 whereas two garages jointly contribute \$12,489.25 (a contribution which is smaller than the twice of the one garage contribution) to the value of a bungalow. When a garage is attached to the bungalow (criterion 4), its sale price is increased by \$2,723.18. An unfinished basement (criterion 5) does not contribute at all to the value of a bungalow. However, a half-finished or completely finished basement contributes, respectively, \$9,075(=2(15,117.48)-21,160.4) and \$14,063 to the value of a bungalow (see Figure 3). The contribution of a fireplace (criterion 6) is null (the majority of properties in the sample have at least one fireplace) whereas two fireplaces makes a marginal contribution of \$6,390.42 (=2(3,195.21)). See Figure 4. Finally, a bungalow being in a certain residential sector seems to change considerably. The contributions of sectors are \$1,905.86 for sector 1, \$0 for sector 2, \$8,894.8 for sector 3, \$18,244.83 for sector 4, and \$5,589.34 for sector 6.

Now a few words about the explicative power of the estimation model (M_A) are in order. The explicative power of the model will be measured in terms of the differences between the actual and the estimated selling prices that are obtained from the model. The computational results given in Appendix 3 will be the basis of the remarks to be made. Note that the average deviation rate in absolute values μ_A is approximately 5% whereas the minimum and maximum deviation rates are respectively 0% and 28%. It can also be observed that 92 bungalows out of 108 are associated with deviations which are smaller than or equal to 10%, 69 Bungalows with deviations smaller than or equal to 5%. For 9 bungalows, this deviation percentage is between 10% and 15%. For only 7 bungalows, this rate is greater than 15%.

If the parameter estimates obtained from the variant (M_A) are used for valuation of bungalows the error to be committed will be around 5% on the average, a level of error that might be acceptable if a large pool of bungalows is to be considered. However, one should be cautious if there are only a few bungalows for valuation, since one might face the risk of under or over estimation above 15%. The second variant (M_B) could be a way out in such cases.

The use of the variant (M_B) requires, at first, the determination of e, the acceptable maximum deviation rate (in absolute value). After solving problem (P) we obtain 14.05% as the value of $\underline{e}$. This indicates, within the context of the empirical study, that the absolute value for the least highest maximum deviation rate is 14.05%. By fixing e to this value, the variant (M_B) was run to obtain the estimates for the parameters of bungalow market behavior. The results are given in Appendices 4 and 5.

The average deviation rate (in absolute value) μ_B climbs, in the case of (M_B), from 5% to 7%. The difference between μ_B and μ_A represents the consequence of having a constraint imposed on the deviation rate (in absolute value) between 0 and 14.05%.

In practice, however, one can minimize the negative effect of $\underline{e}$ by excluding from the sample those bungalows that exhibit a high deviation rate and some distinctiveness with respect to the others. For the empirical study, a visit to 7 properties associated with a deviation rate (in absolute value) greater than 15% revealed, in each case, the existence of one or more unusual characteristics not reflected by the criteria used, such as the proximity of a highway, the necessity to make major repairs at the time of the sale, etc. These properties may be dismissed if such omissions lead to decreases in the values of μ_A, μ_B and e and if the new reduced sample still represents adequately the target population.

The instability of the estimates obtained from the estimation models suggested in this paper can be studied and guided with a post-optimality analysis as proposed by Jacquet-Lagrèze and Siskos [5] and Siskos [14]. It consists of imposing a constraint on the objectives of linear programs, that is, by increasing its optimal value by a small ε after the solution.

6. Conclusion

The estimation models proposed in this paper are only tools to help those socio-economic agents who are concerned with or involved in real estate evaluation. There are several future research avenues one can pursue. One of them is to develop interactive decision support systems to effectively and efficiently implement the proposed methodology as well as other models, including statistical approaches. Development of such computer-aided systems will permit the actors in real estate markets and all other socio-economic agents to follow, in real time, the markets closely and will facilitate the decision-making for potential users.

One of the major difficulties in producing estimates for real estate prices is to decide on which set of attributes or criteria to be used in the evaluation process. In fact, the number of pertinent criteria is generally large, and often, it is difficult to judge *a priori* their relative importance. Of course, we can consider them all in the estimation method, but in that case one is taking a risk of obtaining unreliable estimates from the estimation models. To overcome such inconveniences, below we propose an approach.

Let λ be the maximum number of criteria to be included in the estimation model, and α_j a binary variable which is equal to 1, if the criterion j is included, and 0 otherwise. Redefining the set C as being the set of all the conceivable pertinent criteria. In order to impose a maximum number of criteria that will be included in the estimation method, we can add the following constraints to the variants (M_A) or (M_B) as well as to problem (P):

$$\sum_{t=1}^{T} (a_{tj} + W_{tj}) - M\alpha_j \leq 0, \quad \text{for } j \in C \tag{19}$$

$$\sum_{j \in C} \alpha_j \leq \lambda \tag{20}$$

$$\alpha_j = 0 \text{ or } 1, \quad \text{for } j \in C. \tag{21}$$

where M is some value greater than or equal to $\sum_{t=1}^{T} (a_{tj} + W_{tj}), \quad \forall j \in C.$

Observe that, if α_j is equal to 0, the constraint (19) imposes that the marginal contribution of the criterion j is null for any possible interval. Otherwise, this constraint is inactive. Constraint (20), imposes that the maximum number of criteria, for which the associated constraints in (19) are inactive, will be at the most equal to λ. Because of (21), the variants (M_A) and (M_B) as well as problem (P) become mixed integer linear programming problems.

Appendix 1. Empirical Study Data

i	1	2	3	4	5	6	$*$	p_i	i	1	2	3	4	5	6	$*$	p_i
1	24	1057	0	0	2	0	1	84000	55	16	1138	2	0	3	1	5	105000
2	24	1031	2	0	3	1	2	103000	56	15	1293	2	0	2	1	5	113900
3	23	1337	2	0	3	0	2	94000	57	20	1140	2	0	3	1	1	97000
4	27	957	1	1	3	0	3	90000	58	22	1146	2	0	3	0	2	106000
5	24	1164	2	0	3	0	3	102000	59	21	1440	2	0	3	1	1	121500
6	27	1052	1	1	3	0	3	100000	60	21	1310	2	0	3	2	1	119000
7	26	1079	2	0	3	1	3	104500	61	24	1114	2	0	3	0	3	107500
8	25	1070	2	0	3	0	3	91000	62	23	1047	2	0	1	0	1	79100
9	23	900	2	0	3	0	2	95000	63	21	966	1	0	3	0	1	104500
10	15	1277	2	0	3	1	1	123000	64	14	1484	2	1	3	2	5	143000
11	24	1085	2	0	3	1	1	104500	65	15	1230	2	1	3	2	5	130000
12	24	1086	2	0	2	0	4	110000	66	15	1279	2	1	3	2	5	127000
13	23	1162	1	1	2	0	2	95000	67	21	1270	2	0	2	1	1	108000
14	25	1104	2	0	3	0	3	114000	68	23	1100	2	0	3	0	2	106500
15	27	966	2	0	3	1	3	106500	69	22	1280	2	0	3	1	3	105000
16	25	1177	2	0	3	0	3	119900	70	23	1225	2	0	1	0	3	106000
17	27	994	1	0	2	0	3	93000	71	22	1145	2	0	3	2	1	108000
18	22	1130	2	0	2	0	1	98500	72	15	1381	2	1	2	1	5	118000
19	18	1319	2	1	2	2	4	135000	73	17	1100	2	0	1	2	5	95000
20	23	1089	1	0	3	0	5	105000	74	21	1040	2	0	3	1	1	96000
21	21	1151	2	1	3	1	5	118500	75	21	1218	2	0	3	0	2	105000
22	19	1083	2	0	3	0	5	106900	76	23	1176	2	0	3	0	3	108000
23	17	1144	2	1	2	1	5	110000	77	24	1105	1	1	3	1	3	105000
24	18	1226	2	0	1	0	5	102000	78	21	1142	2	0	3	0	3	105000
25	16	1114	2	1	3	0	5	109000	79	19	1434	0	0	2	1	1	104000
26	16	1230	2	1	3	2	5	138500	80	22	1147	2	0	3	1	1	97000
27	16	1340	2	1	3	0	5	144500	81	23	1400	2	0	3	2	3	110000
28	21	1174	2	0	3	2	5	118500	82	14	1360	2	1	3	1	5	120000
29	22	956	2	0	2	1	5	93000	83	15	1468	2	1	1	0	5	138000
30	20	1080	2	0	3	0	5	102000	84	14	1351	2	0	3	0	5	116500
31	21	1207	2	0	3	0	5	125000	85	17	1120	1	1	3	0	5	98000
32	22	1272	2	1	3	1	4	133000	86	24	1213	2	0	3	1	1	103500
33	23	1250	2	0	3	0	1	89000	87	21	1040	2	0	3	0	2	84000
34	25	1330	2	1	3	0	3	116500	88	22	929	2	0	2	1	2	97000
35	15	1070	1	1	2	0	5	87500	89	23	1118	1	1	3	0	3	112000
36	17	1295	1	0	3	1	5	133300	90	23	976	2	0	2	0	2	88500
37	22	1019	2	0	3	0	5	108000	91	27	985	2	0	2	0	3	102000
38	21	1142	2	0	3	0	5	105000	92	15	1337	2	1	3	2	5	125500
39	17	1215	2	0	2	1	5	110000	93	15	1046	1	1	2	1	5	101500

* k such as $V_{i,k+6} = 1$

Appendix 1. Continued

i	1	2	3	4	5	6	$*$	p_i	i	1	2	3	4	5	6	$*$	p_i
40	16	1137	2	0	3	2	5	122000	94	17	990	2	0	3	0	5	103500
41	23	974	2	0	3	0	1	100000	95	21	1120	0	0	2	0	1	91000
42	23	900	0	0	2	0	4	91500	96	21	1208	2	0	3	1	2	89900
43	22	1054	2	0	3	0	2	91500	97	25	966	2	0	3	1	4	88000
44	24	1111	2	0	2	1	4	113000	98	26	1439	0	0	2	2	3	91000
45	21	1070	2	0	3	1	2	94000	99	25	1140	2	0	3	1	3	116000
46	27	1130	0	0	2	0	3	89000	100	22	1080	2	0	2	0	1	88500
47	28	1008	1	0	3	0	3	101000	101	16	1093	2	0	3	1	5	91000
48	22	1154	2	0	2	0	1	97500	102	22	1430	2	0	3	1	1	119000
49	24	1226	2	0	3	0	2	112500	103	25	1177	1	0	1	0	4	105000
50	23	1263	2	0	3	0	2	110000	104	23	1244	2	0	3	0	2	119900
51	20	1442	2	0	3	0	5	114000	105	26	1192	2	0	3	0	3	116000
52	20	1202	2	0	3	0	1	113500	106	24	1419	2	0	3	0	3	127500
53	21	1070	2	0	3	0	2	106000	107	24	1193	2	0	3	2	4	127500
54	26	1149	2	0	3	2	3	106500	108	15	1120	1	1	3	0	5	104000

* k such as $V_{i,k+6} = 1$

Appendix 2. Values of the estimated parameters with the model (M_A)

$U_o = 70365.04$		Interval t				
		1	2	3	4	5
Criterion 1.	W_{t1}	0.00	0.00	0.00	0.00	0.00
	a_{t1}	0.00	0.00	−2857.57	−2857.57	−3724.82
Criterion 2.	W_{t2}	0.00	38.78	97.69	0.00	131.79
	a_{t2}	0.00	−39436.20	−106208.60	15939.96	−164240.10
Criterion 3.	W_{t3}	0.00	0.00	0.00		
	a_{t3}	0.00	8390.42	12489.25		
Criterion 4.	W_{t4}	0.00	0.00			
	a_{t4}	0.00	2723.18			
Criterion 5.	W_{t5}	0.00	15117.48	0.00		
	a_{t5}	0.00	−21160.40	14063.33		
Criterion 6.	W_{t6}	0.00	0.00	3195.21		
	a_{t6}	0.00	0.00	0.00		
Criterion 7.	W_{t7}	0.00				
	a_{t7}	1905.86				
Criterion 8.	W_{t8}	0.00				
	a_{t8}	0.00				
Criterion 9.	W_{t9}	0.00				
	a_{t9}	8894.80				
Criterion 10.	W_{t10}	0.00				
	a_{t10}	18244.83				
Criterion 11.	W_{t11}	0.00				
	a_{t11}	5589.34				

Appendix 3. Analysis of the predictive performances of the model (M_A)

i	p_i	$U_0 + \sum_{j \in C} U_{ij}$	$(d_i^+ - d_i^-)/p_i$	i	p_i	$U_0 + \sum_{j \in C} U_{ij}$	$(d_i^+ - d_i^-)/p$
1	84000	80047.03	−0.05	55	105000	107466.83	0.02
2	103000	94610.79	−0.08	56	113900	113458.15	−0.00
3	94000	110000.01	0.17	57	97000	101121.15	0.04
4	90000	100711.95	0.12	58	106000	99801.42	−0.06
5	102000	110454.60	0.08	59	121500	121499.95	−0.00
6	100000	102077.17	0.02	60	119000	118296.28	−0.01
7	104500	104500.00	0.00	61	107500	106724.71	−0.01
8	91000	105018.19	0.15	62	79100	83073.87	0.05
9	95000	94060.05	−0.01	63	104500	91867.07	−0.12
10	123000	114763.43	−0.07	64	143000	142953.26	−0.00
11	104500	98611.O1	−0.06	65	130000	125567.69	−0.03
12	110000	110000.00	0.00	66	127000	127560.52	0.00
13	95000	95000.01	0.00	67	108000	106917.09	−0.01
14	114000	106336.87	−0.07	68	106500	97286.93	−0.09
15	106500	102087.60	−0.04	69	105000	118894.81	0.13
16	119900	111724.54	−0.07	70	106000	102350.21	−0.03
17	93000	93000.00	0.00	71	108000	108000.00	0.00
18	98500	95367.55	−0.03	72	118000	117999.94	−0.00
19	135000	132369.67	−0.02	73	95000	98060.93	0.03
20	105500	101208.38	−0.04	74	96000	96865.71	0.01
21	118500	108602.38	−0.08	75	105000	106834.93	0.02
22	106900	105074.50	−0.02	76	108000	111625.85	0.03
23	110000	105787.37	−0.04	77	105000	105000.00	0.00
24	102000	102000.00	0.00	78	105000	108305.47	0.03
25	109000	109000.00	0.00	79	104000	106088.77	0.02
26	138500	125567.69	−0.09	80	97000	101840.97	0.05
27	144500	121170.10	−0.16	81	110000	129607.81	0.18
28	118500	114516.43	−0.03	82	120000	121170.10	0.01
29	93000	94660.61	0.02	83	138000	120390.90	−0.13
30	102000	102100.57	0.00	84	116500	118446.92	−0.13
31	125000	111349.70	−0.11	85	98000	105133.88	0.07
32	133000	130968.02	−0.02	86	103500	108252.35	0.05
33	8900	111866.79	0.26	87	84000	94959.85	0.13
34	116500	121617.99	0.04	88	97000	89071.28	−0.08
35	87500	98205.88	0.12	89	112000	105504.20	−0.06
36	133300	114348.09	−0.14	90	88500	89071.28	0.01
37	108000	99734.71	−0.08	91	102000	97098.83	−0.05
38	105000	105000.00	0.00	92	125500	127560.52	0.02
39	110000	110000.00	0.00	93	101500	97275.05	−0.04
40	122000	113759.57	−0.07	94	103500	102506.96	−0.01
41	100000	95965.90	−0.04	95	9100	82490.46	−0.09

Appendix 3. Continued

i	p_i	$U_0 + \sum_{j \in C} U_{ij}$	$(d_i^+ - d_i^-)/p_i$	i	p_i	$U_0 + \sum_{j \in C} U_{ij}$	$(d_i^+ - d_i^-)/p$
42	91500	94826.86	0.04	96	89900	105858.05	0.18
43	91500	95502.84	0.04	97	88000	112304.88	0.28
44	113000	110969.62	−0.02	98	91000	91000.00	0.00
45	94000	96123.39	0.02	99	116000	108110.10	−0.07
46	89000	89000.00	0.00	100	88500	93428.32	0.06
47	101000	97988.77	−0.03	101	91000	105462.35	0.16
48	97500	97500.00	0.00	102	119000	120182.07	0.01
49	112500	107616.43	−0.04	103	105000	102912.41	−0.02
50	110000	110000.00	0.00	104	119900	109374.81	−0.09
51	114000	125447.00	0.10	105	116000	112322.61	−0.03
52	113500	107177.78	−0.06	106	127500	125721.35	−0.01
53	106000	96123.39	−0.09	107	127500	129027.99	0.01
54	106500	114512.46	0.08	108	104000	105133.88	0.01

References

1. Beckwith, N.E. and D.R. Lehmann (1973), The Importance of Differential Weights in Multiples Attribute Models of Consumer Attitude, *Journal of Marketing Research* **10**, 141–145.
2. Caroll, J.D. (1972), Individual Differences and Multi-Dimensional Scaling, In Shepard, R.N. et al. *Multidimensional Scaling: Theory and Applications in Behavioral Sciences*, Vol. I, Seminar Press, New York.
3. Green, P.E. and V.R. Rao (1971), Conjoint Measurement for Qualifying Judgmental Data, *Journal of Marketing Research* **8**, 355–363.
4. Horsky, D. and M.R. Rao (1984), Estimation of Attribute Weights from Preference Comparison, *Management Science* **30**, 801–822.
5. Jacquet-Lagrèze, E. and Y. Siskos (1982), Assessing a Set of Additive Utility Functions for Multicriteria Decision-Making, the UTA Method, *European Journal of Operational Research* **10**, 151–164.
6. Jain, A.K., Actio, N.K. Malhotra and V. Mahajan (1979), A Comparison of the Internal Validity of Alternative Parameter Estimation Methods in Decompositional Multi-Attribute Preference Models, *Journal of Marketing Research* **16**, 313–322.
7. Keeney, R.L. and H. Raïffa (1976), *Decision with Multiple Objectives: Preferences and Value Trade-offs*, Wiley, N.Y.
8. Kettani, O, (1988), *Nouvelles formulations des problèmes de choix multicritère basée sur la programmation mathématique*, Doctorate thesis, Université Laval, Québec, Canada.
9. Oral, M. and O. Kettani (1989), A Mathematical Programming Model for Market Share Prediction, *International Journal of Forecasting* **5**, 59–68.
10. Oral, M. and O. Kettani (1989), Modeling the Process of Multiattribute Choice, *Journal of the Operational Research Society* **40**, 101–111.
11. Oral, M. and O. Kettani (1988), Modélisation des parts de marché: une approche multicritère, *Revue Canadienne des Sciences de l'Administration* **5**, 40–50.
12. Oral, M., O. Kettani and M. Daouas (1993), A Methodology for Country Risk Rating, *Transactions on Operational Research* **5**, 1–17.
13. Pekelman, D. and K.S. Subrata (1974), Mathematical Programming Models for the Determination of the Attribute Weights, *Management Science* **20**, 1217–1229.

14. Siskos, J. (1985), Analyses de régression et programmation linéaire, *Revue de Statistique Appliquée* XXXIII, 41–45.
15. Siskos, J. and C. Zopounidis (1987), The Evaluation Criteria of the Venture Capital Investment Activity: An Interactive Assessment, *European Journal of Operational Research* **31**, 304–313.
16. Shocker, A.D. and V. Srinivasan (1979), Multiattribute Approaches for Product Concept Evaluation and Generation: A Critical Review, *Journal of Marketing Research* **16**, 159–180.
17. Stewart, T.J. (1990), A Critical Survey on the Status of Multiple Criteria Decision Making Theory and Practice, *Omega* **20**, 569–586.
18. Srinivasan, V. and A.D. Shocker (1973), Estimating the Weights of Multiple Attributes in a Composite Criterion Using Pairwise Judgements, *Psychometrica* **38**, 472–493.
19. Srinivassan, V. and A.D. Shocker (1973), Linear Programming Techniques for Multidimensional Analysis of Preferences, *Psychometrica* **38**, 337–339.